Grades 8 - 9 PROMPT sheet

A/1 Use fractional & negative indices

- **Rules when working with indices:**

$a^x \times a^y = a^{(x+y)}$

$a^3 \times a^2 = a^{(3+2)} = a^5$

$2^3 \times 2^2 = 2^{(5)} = 32$

$a^x \div a^y = a^{(x-y)}$

$a^7 \div a^3 = a^{(7-3)} = a^4$

$3^7 \div 3^3 = 3^{(4)} = 81$

$(a^x)^y = a^{(x\,y)}$

$(a^3)^2 = a^6$

$(2^3)^2 = 2^6 = 64$

$a^0 = 1$

$y^0 = 1$

$8^0 = 1$

$a^{-x} = \dfrac{1}{a^x}$

$a^{-3} = \dfrac{1}{a^3}$

$2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

$a^{x/y} = (\sqrt[y]{a})^x$

$a^{2/5} = (\sqrt[5]{a})^2$

$32^{2/5} = (\sqrt[5]{32})^2 = 2^2$

$a^{-x/y} = \dfrac{1}{(\sqrt[y]{a})^x}$

A/2 Manipulate and simplify surds

$\sqrt{25}$ is NOT a surd because it is exactly 5

$\sqrt{3}$ is a surd because the answer is not exact

A surd is an irrational number

- **To simplify surds look for square number factors**

$\sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

- Rules when working with surds:

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{3} \times \sqrt{15} = \sqrt{45} = \sqrt{9x5} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

$m\sqrt{a} + n\sqrt{a} = (m+n)\sqrt{a}$

$2\sqrt{5} + 3\sqrt{5} = (2+3)\sqrt{5} = 5\sqrt{5}$

$\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

Square number

$\sqrt{\dfrac{72}{20}} = \dfrac{\sqrt{72}}{\sqrt{20}} = \dfrac{\sqrt{36} \times \sqrt{2}}{\sqrt{4} \times \sqrt{5}} = \dfrac{6\sqrt{2}}{2\sqrt{5}} = \dfrac{3\sqrt{2}}{\sqrt{5}}$

Square number

- **Rationalising the denominator**

This is the removing of a surd from the denominator of a fraction by multiplying both the numerator & the denominator by that surd

In general:

$\dfrac{a}{\sqrt{b}} =$

$= \dfrac{a}{\sqrt{b}} \times \dfrac{\sqrt{b}}{\sqrt{b}}$ (Multiply both top & bottom by √b)

$= \dfrac{a\sqrt{b}}{b}$

Example

$\dfrac{6}{\sqrt{12}}$

$= \dfrac{6}{\sqrt{12}} \times \dfrac{\sqrt{12}}{\sqrt{12}}$ (Multiply both top & bottom by √12)

$= \dfrac{6\sqrt{12}}{12} = \dfrac{\sqrt{12}}{2} = \dfrac{\sqrt{4} \times \sqrt{3}}{2} = \dfrac{2\sqrt{3}}{2} = \sqrt{3}$

A/3 Upper & lower bounds

- **If 'a' is rounded to nearest 'x'**

Upper bound = a + $\frac{1}{2}$x

Lower bound = a – $\frac{1}{2}$x

e.g. if 1.8 is rounded to 1dp

Upper bound = 1.8 + $\frac{1}{2}$(0.1) = 1.85

Lower bound = 1.8 – $\frac{1}{2}$(0.1) = 1.75

- **Calculating using bounds**

Adding bounds

Maximum = Upper + upper

Minimum = Lower + lower

Subtracting bounds

Maximum = Upper - lower

Minimum = Lower – upper

Multiplying

Maximum = Upper x upper

Minimum = Lower x lower

Dividing

Maximum = Upper ÷ lower

Minimum = Lower ÷ upper

A/4 Direct and inverse proportion

The symbol $\propto$ means:
'varies as' or 'is proportional to'

- **Direct proportion**

If: $y \propto x$ or $y \propto x^2$ or $y \propto x^3$

Formulae: $y = kx$ or $y = kx^2$ or $y = kx^3$

Example

y is directly proportional to x
When y = 21, then x = 3
(find value of k first by substituting these values)

$$y \propto x \quad \therefore y = kx$$
$$21 = k \times 3$$
$$\therefore \underline{k = 7}$$
$$\underline{y = 7x}$$

(Now this equation can be used to find y, given x)

- **Inverse proportion**

If: $y \propto \frac{1}{x}$ or $y \propto \frac{1}{x^2}$ or $y \propto \frac{1}{x^3}$

Formulae: $y = \frac{k}{x}$ or $y = \frac{k}{x^2}$ or $y = \frac{k}{x^3}$

Example

a is inversely proportional to b
When a = 12 and b = 4

$$a \propto \frac{1}{b} \quad \therefore a = \frac{k}{b}$$
$$12 = \frac{k}{4}$$
$$\therefore k = 48$$
$$\therefore a = \frac{48}{b}$$

A/5 Solve quadratic equation by factorising

- **Put equation in form $ax^2 + bx + c = 0$**

$2x^2 - 3x - 5=0$

- **Factorise the left hand side**

$(2x - 5)(x + 1) = 0$

- **Equate each factor to zero**

$2x - 5 = 0$ or $x + 1 = 0$

$\underline{\mathbf{x = 2.5 \text{ or } x = -1}}$

A/6 Solve quadratic equations by formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

To solve: $3x^2 + 4x - 2 = 0$

$a = 3$
$b = 4$
$c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{16+24}}{6}$$

$$= \frac{-4 \pm \sqrt{40}}{6}$$

$$x = \frac{-4 + \sqrt{40}}{6} \quad \text{OR} \quad \frac{-4 - \sqrt{40}}{6}$$

$\underline{x = 0.39\text{(2dp)} \quad \text{OR} \quad -1.72 \text{ (2dp)}}$

A/7 Solve quadratic equation by completing the square

- **Make the coefficient of x^2 a square**

$2x^2 + 10x + 5 = 0$ (mult by 2)
⇨ $4x^2 + 20x + 10 = 0$

- **Add a number to both sides to make a perfect square**

$4x^2 + 20x + 10 = 0$ (Add 15)
$4x^2 + 20x + 25 = 15$
⇨ $(2x + 5)^2 = 15$

- **Square root both sides**

$2x + 5 = \pm\sqrt{15}$ (-5 from both sides)
$2x = -5 \pm \sqrt{15}$
$x = \frac{-5 + \sqrt{15}}{2}$ OR $\frac{-5 - \sqrt{15}}{2}$
$\underline{x = -0.56 \quad \text{OR} \quad -4.44 \text{ (2dp)}}$

A/8 Simplify algebraic fractions

Adding & subtracting algebraic fractions

Example 1

$\frac{x + 3}{4} + \frac{x - 5}{3}$ (common denominator is 12)

$= \frac{3(x + 3)}{12} + \frac{4(x - 5)}{12}$

$= \frac{3x + 9 + 4x - 20}{12}$

$= \frac{7x - 11}{12}$

Example 2

$\frac{5}{(x + 1)} - \frac{3}{(x + 2)}$ (common denominator is (x+1)(x+2)

$= \frac{5(x + 2) - 3(x + 1)}{(x+1)(x+2)}$

$= \frac{5x + 10 - 3x - 3}{(x+1)(x+2)}$

$= \frac{2x + 7}{(x+1)(x+2)}$

- **Simplifying algebraic fractions**

Example

$\frac{2x^2 + 3x + 1}{x^2 - 3x - 4}$ (factorise)

$= \frac{(2x + 1)\cancel{(x + 1)}}{(x - 4)\cancel{(x + 1)}}$ (cancel)

$= \frac{(2x + 1)}{(x - 4)}$

A/9 Solve equations with fractions

$\frac{x}{2x - 3} + \frac{4}{x + 1} = 1$ Common denominator (2x-3)(x+1)

$\frac{x(x+1) + 4(2x-3)}{(2x-3)(x+1)} = 1$

$\frac{x^2 + x + 8x - 12}{(2x-3)(x+1)} = 1$

$x^2 + 9x - 12 = 1(2x-3)(x+1)$

$x^2 + 9x - 12 = 2x^2 - x - 3$ ($-x^2$ from both sides)

$9x - 12 = x^2 - x - 3$ (-9x from each side)

$-12 = x^2 - 10x - 3$ (+12 to each side)

$0 = x^2 - 10x + 9$ (factorise)

$(x + 9)(x + 1) = 0$

$x = -9$ or $x = -1$

A/10 Solve simultaneous equations ~ one is a quadratic

- **Rewrite the linear with one letter in terms of the other**
- **Substitute the linear into the quadratic**

Example

$x + y = 4$ (find one letter in terms of the other

$\Rightarrow y = 4 - x$

$x^2 + y^2 = 40$ (substitute y=4 -x)

$x^2 + (4-x)^2 = 40$ (Expand $(4-x)^2$)

$x^2 + 16 - 8x + x^2 = 40$

$2x^2 - 8x + 16 = 40$ (-40 from each side)

$2x^2 - 8x - 24 = 0$ (÷2 both sides)

$x^2 - 4x - 12 = 0$ (factorise)

$(x - 6)(x + 2) = 0$

$x = 6$ or $x = -2$

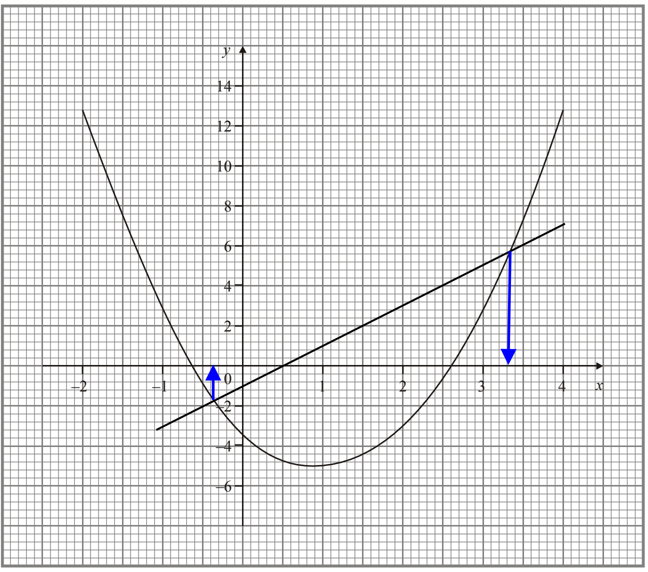
A/10 Solve GRAPHICALLY simultaneous equations ~ one is a quadratic

- **Draw the two graphs and find where they intersect**

Example

$y=2x^2-4x-3$

$y=2x-1$



Solutions are x = -0.3 and x = 3.3

(points of intersection)

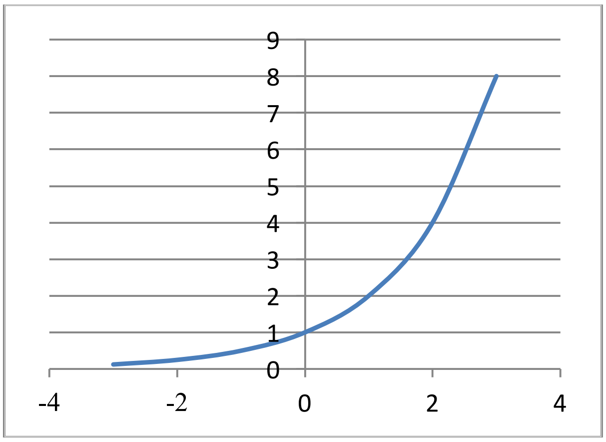
- Sometimes the equation has to be adapted~ rearrange the equation to solve so that the equation of the graph drawn is on the left. On the right is the other equation to be drawn

A/11 Graph of Exponential function

The graph of the exponential function is:

$$y = a^x$$

Example $y = 2^x$



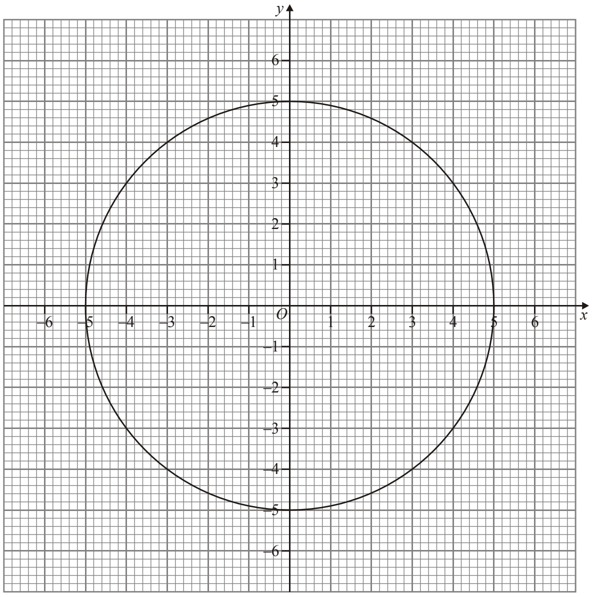
It has no maximum or minimum point
It crosses the y-axis at (0,1)
It never crosses the x-axis

A/12Graph of the circle

The graph of a circle is of the form:

$$x^2 + y^2 = r^2$$

where r is the radius and the centre is (0,0)



This a circle of radius 5 and a centre (0,0)
The graph of this circle is

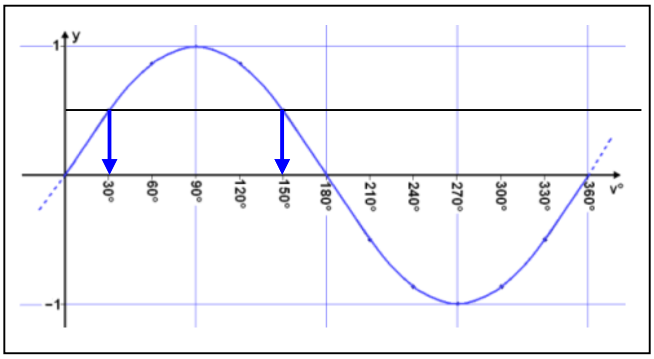
$$x^2 + y^2 = 5^2$$

$$\Rightarrow \quad x^2 + y^2 = 25$$

A/13 Graphs of trigonometric functions

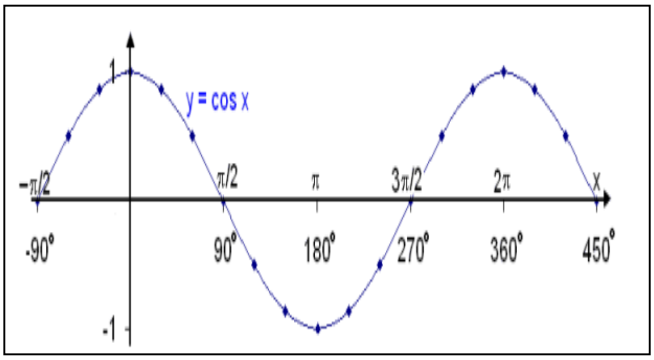
LEARN THE SHAPES OF THE GRAPHS

Graph of y=sin x



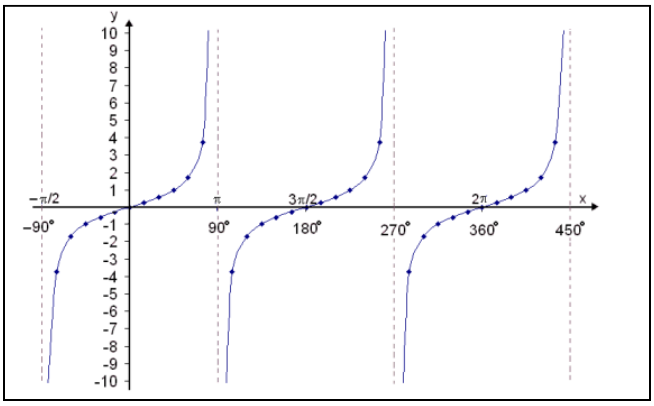
$$-1 \leq \sin x \leq 1$$

Graph y = cos x



$$-1 \leq \cos x \leq 1$$

Graph y = tan x



Tan x is undefined at 90^0, 270^0 ....

Solutions to trigonometrical equations can be found on the calculator and by using the symmetry of these graphs

Example:
If sin x = 0.5
$x = 30^0$, 150^0 , *(See the solutions on sin graph above* or from calculator)

A/14 Transformation of functions

f(x) means 'a function of x'

e.g. $f(x) = x^2 - 4x + 1$

f(3) means work out the value of f(x) when x = 3

e.g. $f(3) = 3^2 - 4x3 + 1 = -2$

In general for any graph y = f(x) these are the transformations

$y = f(x) + a$	Translation $\begin{pmatrix} 0 \\ a \end{pmatrix}$
$y = f(x+ a)$	Translation $\begin{pmatrix} -a \\ 0 \end{pmatrix}$
$y = -f(x)$	Reflection in the x-axis
$y = f(-x)$	Reflection in the y-axis
$y = af(x)$	Stretch from the x-axis Parallel to the y-axis Scale factor=a
$y = f(ax)$	Stretch from the y-axis Parallel to the x-axis Scale factor= $\frac{1}{a}$

A/15 Change the subject of a formula

- **The subject may only appear once**

Use balancing to isolate the new subject

Example : To make 'x' the new subject

$A = \frac{k(x + 5)}{3}$ (multiply both sides by 3)

⇨ $3A = k(x + 5)$ (Expand the bracket)

⇨ $3A = kx + 5k$ (-5k from both sides)

$3A - 5k = kx$ (÷ k both sides)

$\frac{3A - 5k}{k} = \frac{\cancel{k}x}{\cancel{k}}$

$x = \frac{3A - 5k}{k}$

- **The subject may appear twice**

Collect together all the terms containing the new subject & factorise to isolate it

Example: to make 'b' the new subject

$a = \frac{2 - 7b}{b - 5}$ (multiply both sides by (b – 5)

$a(b - 5) = 2 - 7b$ (Expand the bracket)

$ab - 5a = 2 - 7b$ (+7b to both sides)

$7b + ab - 5a = 2$ (+5a to both sides)

To leave terms in b together

$7b + ab = 2 + 5a$ (factorise the left side)

To isolate b

$\frac{b(7 + a)}{(7 + a)} = \frac{2 + 5a}{(7 + a)}$ (÷(7 + a) both sides)

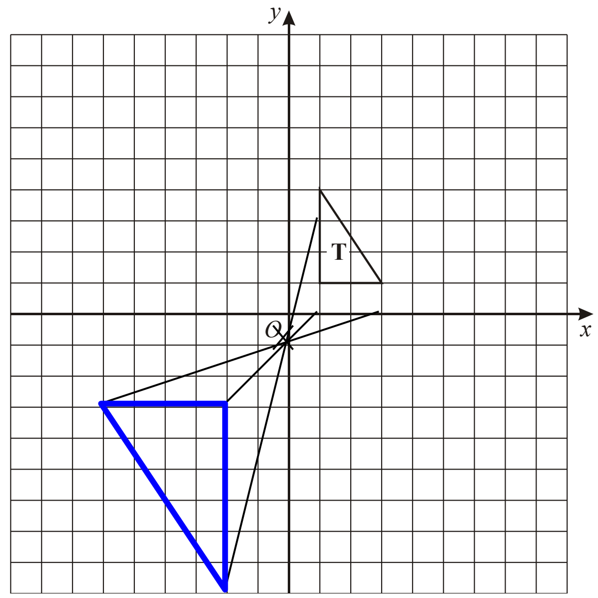
$b = \frac{2 + 5a}{(7 + a)}$

A/16 Enlarge by a negative scale factor

With a negative scale factor:

- The image is on the opposite side of the centre
- The image is also inverted

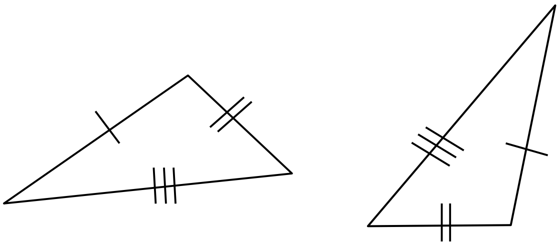
Example : Enlargement scale factor -2 about 0



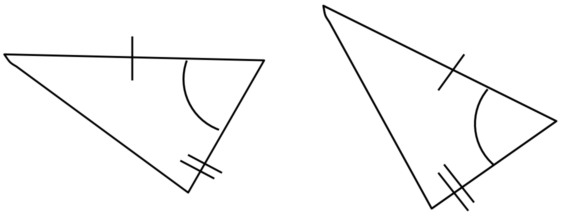
A/17 **Congruence**

- **Congruent shapes have the same size and shape, one will fit exactly over the other.**
- **2 triangles are congruent if any of these 4 conditions are satisfied on each triangle**

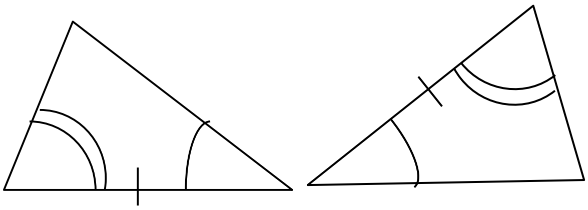
~The corresponding sides are equal ~ **SSS**



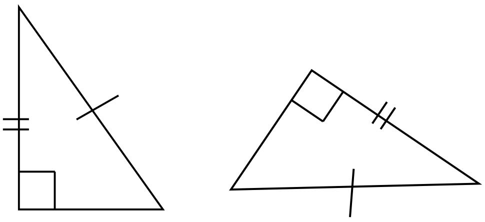
~2 sides & the included angle are equal ~ **SAS**



~2 angles & the corresponding side are equal ~ **ASA**



~Both triangles are right-angled, hypotenuses are equal and another pair of sides are equal ~ **RHS**



A/18 **Similarity & enlargement**

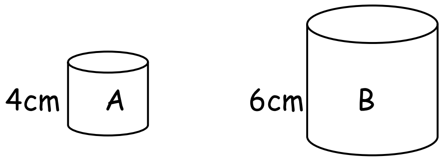
- **For similar shapes when:**

Length scale factor = **k**

Area scale factor = **k^2**

Volume scale factor = **k^3**

Example



If height of A = 4cm & height of B = 6cm

- Length scale factor = 6 ÷ 4 = **1.5**

If surface area of A = 132cm^2

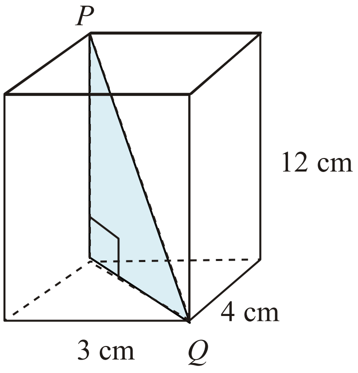
- Surface area of B = 132 x 1.5^2 = 297cm^3

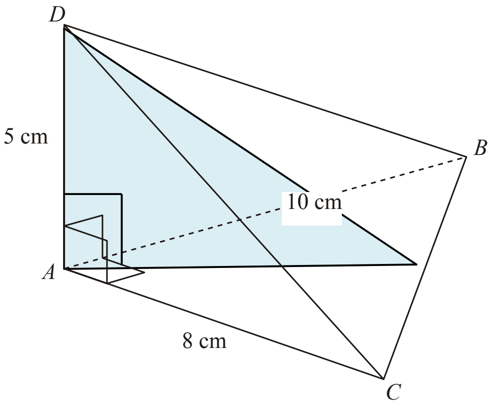
If volume of A = 120 cm^3

- Volume of B = 120 x 1.5^3 = 405cm^3

A/19 **Finding lengths & angles in 3D**

- **Identify the triangle in the 3D shape containing the unknown side/angle**
- **Use Pythagoras and trigonometry as appropriate**





A/20 Sine Rule (non-right angled triangles)

To find an angle use:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

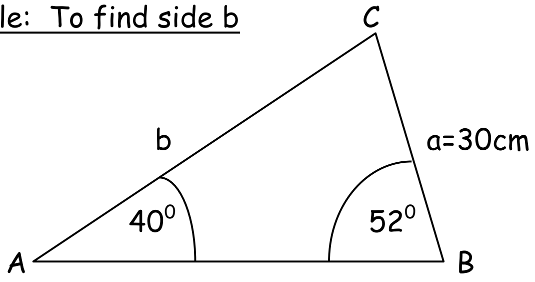
To find a side use:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use SINE RULE when given:

- two sides and a non-included angle
- any two angles and one side

Example: To find side b



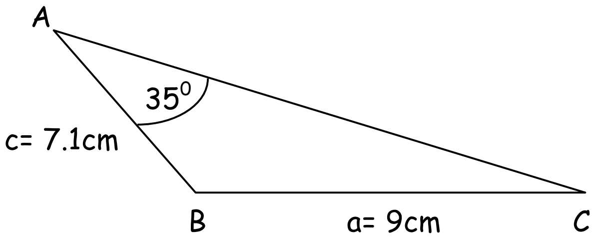
$$\frac{b}{\sin B} = \frac{a}{\sin C}$$

$$\frac{b}{\sin 52^0} = \frac{30}{\sin 40^0}$$

$$b = \frac{30}{\sin 40^0} \times \sin 52^0$$

b = 36.8 cm (1dp)

Example: To find angle C



$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin C}{7.1} = \frac{\sin 35^0}{9}$$

$$\sin C = \frac{\sin 35^0}{9} \times 7.1$$

$\sin C = 0.4524.....$

$C = \sin^{-1}(0.4524.....)$

C = 28.9⁰(1dp)

A/20 Cosine Rule (non-right angled triangles)

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

OR

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

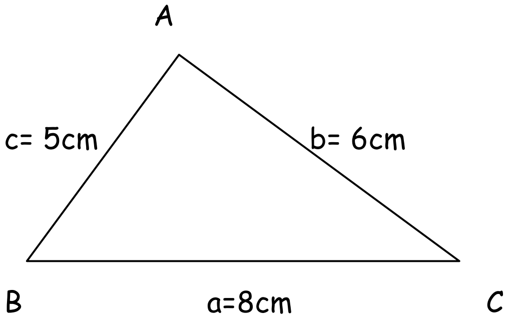
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Use COSINE RULE when given:

- 3 sides
- 2 sides and the included angle

Example: To find angle C



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

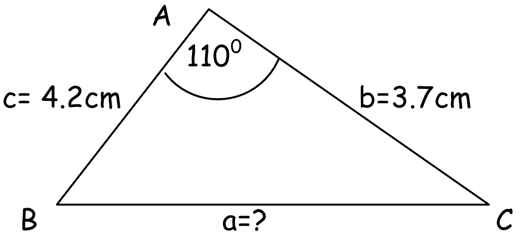
$$\cos C = \frac{8^2 + 6^2 - 5^2}{2x8x6}$$

$\cos C = 0.78125......$

$C = \cos^{-1}(0.78125......)$

C = 38.6⁰(1dp)

Example: To find side a



$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 3.7^2 + 4.2^2 - 2x3.7x4.2 \cos 110^0$

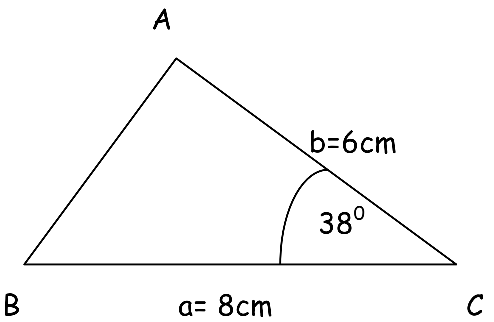
$a^2 = 41.96$

a = 6.48(2dp)

A/20 Area of triangle –height not known

Area = $\frac{1}{2}$ ab sinC
Area = $\frac{1}{2}$ bc sinA
Area = $\frac{1}{2}$ ac sinB

Example



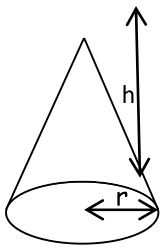
Area = $\frac{1}{2}$ ab sinC
= $\frac{1}{2}$ x 8 x 6 x sin 38^0
= 14.8 cm^2(1dp)

A/21 Pyramid & Sphere –Volume

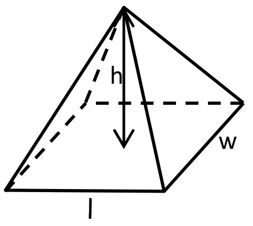
VOLUME - PYRAMID

Volume of Pyramid = $\frac{1}{3}$ x area of cross-section x height

e.g. cone



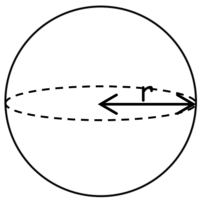
Volume = $\frac{1}{3} \times \pi r^2 h$



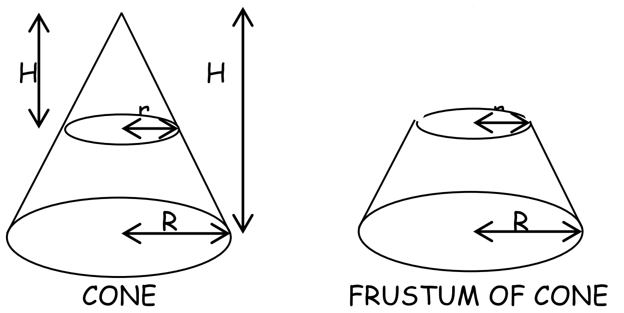
Volume = $\frac{1}{3}$ x l x w x h

VOLUME - SPHERE

Volume of Sphere = $\frac{4}{3}\pi r^3$



VOLUME -FRUSTUM OF A CONE

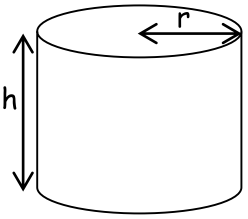


Volume of frustum
= Volume of whole cone – volume of cone removed
= $\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

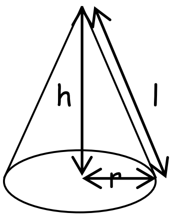
A/21 Pyramid & Sphere – Surface Area

CURVED SURFACE AREA

~Curved surface area of a cylinder = 2πrh

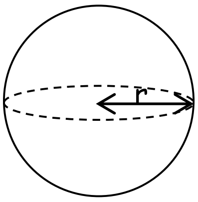


~Curved surface of a cone = πrl

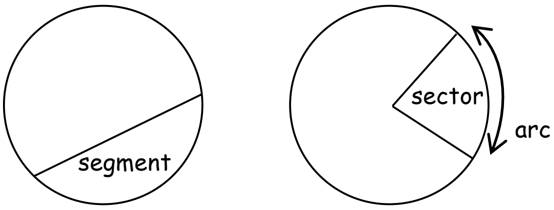


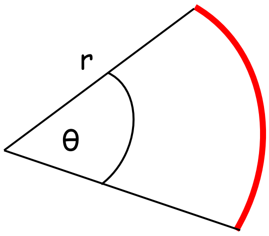
[NB To find 'l' use Pythagoras' Theorem
$l^2 = h^2 + r^2$]

~Curved surface of a sphere = $4\pi r^2$

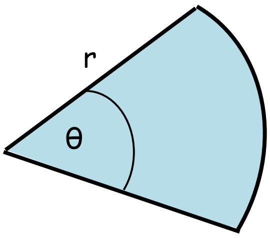


A/22 Length of arc & area of sector





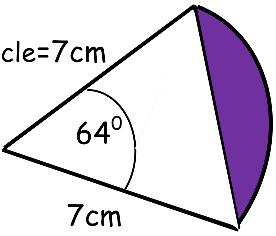
Length of arc = $\frac{\theta}{360^0} \times 2\pi r$



Area of sector = $\frac{\theta}{360^0} \times \pi r^2$

A/22 Area of segment

radius of circle=7cm



Area of segment

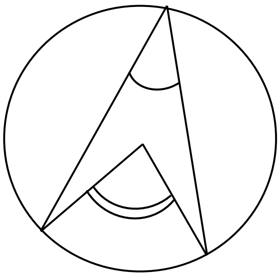
= area of sector – area of triangle

$= \frac{\theta}{360^0} \times \pi r^2 - \frac{1}{2} ab \sin\theta$

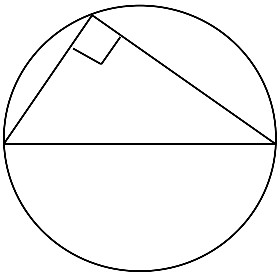
$= \frac{64^0}{360^0} \times \pi \times 7^2 - \frac{1}{2} 7 \times 7 \times \sin 64^0$

$\underline{= 5.35\text{cm}^2 \text{ (1dp)}}$

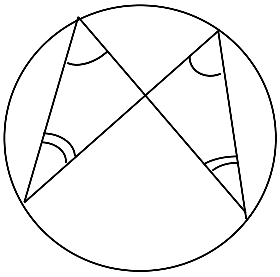
A/23 Circle properties



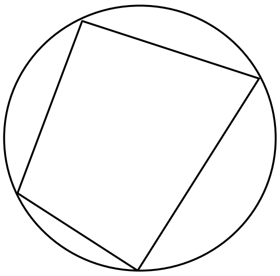
The angle at the centre
= 2 x the angle at the
circumference



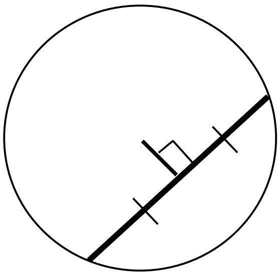
The angle in a semi-circle
is a right angle



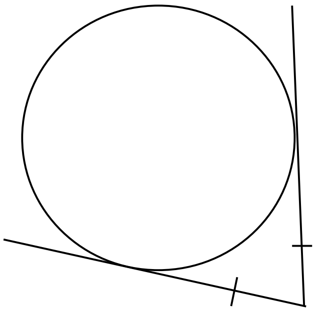
Angles in the same
segment are equal



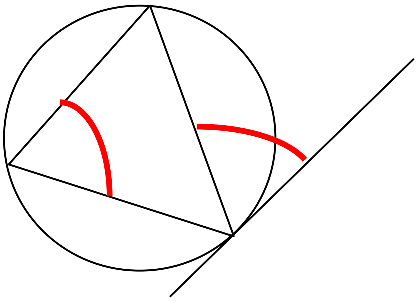
Opposite angles of
a cyclic quadrilateral
add up to 180^0



The perpendicular from
the centre to a chord
bisects the chord



Tangents from a point
to a circle are equal

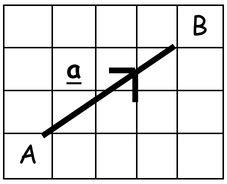


The angle between a tangent and a chord is equal to the angle in the alternate segment

A/24 **Vectors**

- **Vector notation**

This vector can be written as $\begin{pmatrix}3\\2\end{pmatrix}$ or $\underline{a}$ or $\overrightarrow{AB}$

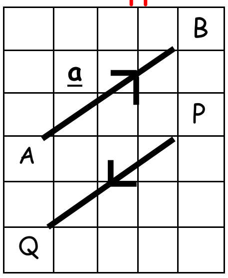


- **A vector has magnitude(length) & direction(shown by an arrow)**

Magnitude can be found by Pythagoras Theorem

$$AB = \sqrt{3^2 + 2^2} = \sqrt{13} = 3.6$$

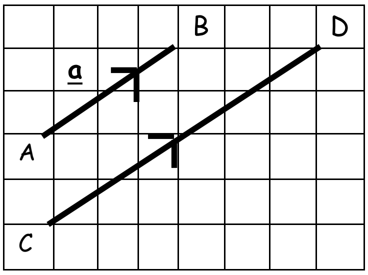
- **A parallel vector with same magnitude but opposite direction**



Vector $\overrightarrow{PQ}$ is equal in length to $\overrightarrow{AB}$ but opposite in direction so we say:

$$\overrightarrow{PQ} = -\underline{a}$$

- **A parallel vector with same direction but different magnitude**



Vector $\overrightarrow{CD}$ is twice (scalar 2) the magnitude but same direction so we say:

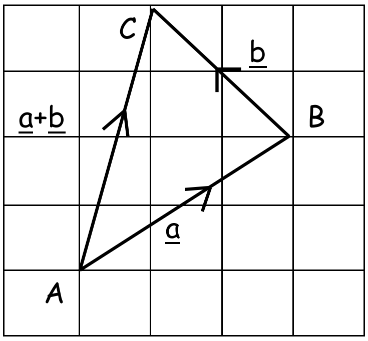
$$\overrightarrow{CD} = 2\underline{a}$$

A negative scalar would reverse the direction

- **Vector addition**

Adding graphically, the vectors go nose to tail

$\underline{a} = \begin{pmatrix}3\\2\end{pmatrix}$ $\underline{b} = \begin{pmatrix}-2\\2\end{pmatrix}$



The combination of these two vectors:

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \underline{a} + \underline{b}$$

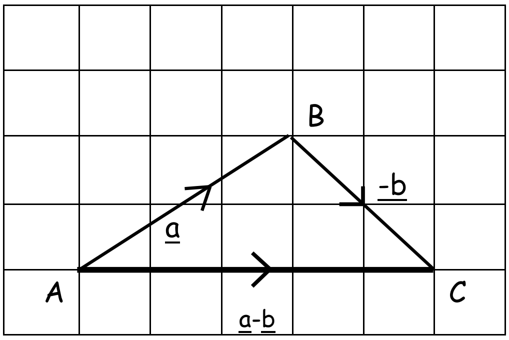
$$\begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}-2\\2\end{pmatrix}$$

$$= \begin{pmatrix}1\\4\end{pmatrix}$$

- **Vector subtraction**

Adding graphically, the vectors go nose to tail

$\underline{a} = \begin{pmatrix}3\\2\end{pmatrix}$ $\underline{b} = \begin{pmatrix}-2\\2\end{pmatrix}$



The combination of these two vectors:

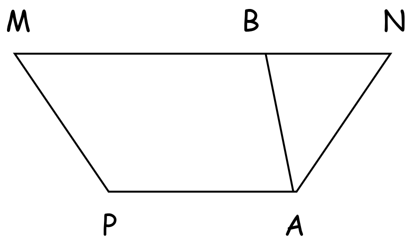
$$\overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AC} = \underline{a} - \underline{b}$$

$$\begin{pmatrix}3\\2\end{pmatrix} - \begin{pmatrix}-2\\2\end{pmatrix}$$

$$= \begin{pmatrix}5\\0\end{pmatrix}$$

$\overrightarrow{AC}$ is called the RESULTANT vector

- **The sum of vectors**



$$\vec{AB} = \vec{AP} + \vec{PM} + \vec{MB}$$

The vector AB is equal to the sum of these vectors or it could be a different route:

go via N

$$\vec{AB} = \vec{AN} + \vec{NB}$$

start point

end point

A/25 Sampling

The sample is:

- a small group of the population.
- an adequate size
- representative of the population

Simple random sampling
Everyone has an equal chance
e.g. pick out names from a hat

Systematic sampling
Arranged in some sort of order
e.g. pick out every 10th one on the list

Stratified sampling
Sample is divided into groups according to criteria
These groups are called strata
A simple random sample is taken from each group in proportion to its size using this formula:

$$\text{No from each group} = \frac{\text{Stratum size}}{\text{Population}} \times \text{Sample size}$$

Example

An inspector wants to look at the work of a stratified sample of 70 of these students.

Language	**Number of students**
Greek	145
Spanish	121
German	198
French	186
Total	**650**

$$\text{No. from Greek} = \frac{145}{650} \times 70 \approx 16$$

$$\text{No. from Spanish} = \frac{121}{650} \times 70 \approx 13$$

$$\text{No. from German} = \frac{198}{650} \times 70 \approx 21$$

$$\text{No. from French} = \frac{186}{650} \times 70 \approx 20$$

This only tells us 'how many' to take - now take a random sample of this many from each language

A/26 Histograms

- Class intervals are not equal
- Vertical axis is the frequency density
- The area of each bar not the height is the frequency

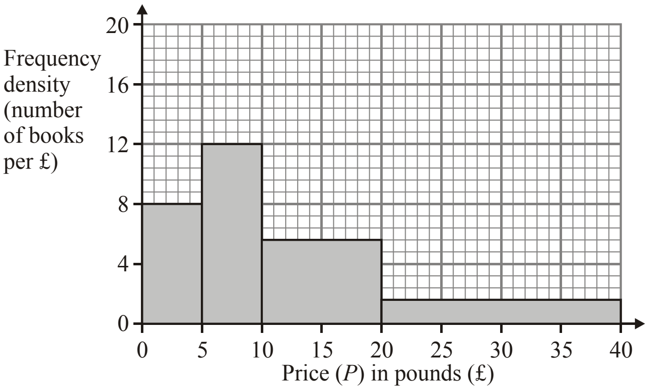
Frequency = class width x frequency density
Frequency density = frequency ÷ class width

To draw a histogram
Calculate the frequency density
Example

Age (*x* years)	Class width	f	Frequency density
$0 < x \le 20$	**20**	**28**	**28**÷**20** = 1.4
$20 < x \le 35$	**15**	**36**	**36**÷**15** = 2.4
$35 < x \le 45$	**10**	**20**	**20**÷**10** = 2
$45 < x \le 65$	**20**	**30**	**30**÷**20** = 1.5

Scale the frequency density axis up to 2.4
Draw in the bars to relevant heights & widths

To interpret a histogram



NOTE: On the vertical axis each small square = 0.8

Price (*P*) in pounds (£)	f = width x height
$0 < P \le 5$	5 x 8 = 40
$5 < P \le 10$	5 x 12 = 60
$10 < P \le 20$	10 x 5.6 = 56
$20 < P \le 40$	20 x 1.6 = 32

A/27 Probability – the 'and' 'or' rule

P(A or B) = p(A) + p(B)
Use this addition rule to find the probability of either of two mutually exclusive events occurring

e.g. p(a 3 on a dice **or** a 4 on a dice)

$$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

P(A and B) = p(A) x p(B)
Use this multiplication rule to find the probability of either of both of two independent events occurring

e.g. p(Head on a coin **and** a 6 on a dice)

$$= \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

A/28 Probability – Tree diagram for successive dependent events

When events are dependent, the probability of the second event is called a conditional event because it is conditional on the outcome of the first event

Example

2 milk and 8 dark chocolates in a box
Kate chooses one and eats it. (ONLY 9 left now)
She chooses a second one
This can be shown on a tree diagram